The Effect of Capture on Collison-Resolution Algorithms

ISRAEL CIDON AND MOSHE SIDI, MEMBER, IEEE

Abstract—**In many communication systems, the stronger of two or more overlapping packets might capture the receiver and thus be received without error. The effect of capture on collision-resolution algorithms in a slotted ALOHA type broadcasting network is investigated here. Extensions to the algorithms are suggested for both the situations in which the receiver can or cannot distinguish between success slots and capture slots. In particular, we present a class of retransmission schemes for packets that have been transmitted during capture slots but have not been received correctly.**

The performance analysis is confined to a simplified model in which the nodes of the network are divided into two groups and only packets sent by the nodes of one of them might be captured. For this simplified model and for each extended algorithm, explicit recursive equations are given, from which the average conditional collision-resolution interval length, as well as the maximal throughput, can be determined. As expected, we show that in the presence of capture, the performance of the network is improved and the maximal attainable throughput is increased.

Extensions of the simplified model such as dividing the nodes into K groups instead of two, or considering the situation that capture depends on relative distances and transmission powers, are also discussed. For the latter situation we give simulation results.

I. Introduction

PREVIOUS studies of collision-resolution algorithms (CRA) in a slotted ALOHA type broadcasting network [1]-[7] use the assumption that whenever two or more packets are transmitted in the same slot, then neither of them is correctly received at the common receiver. In fact, this assumption provides a lower bound to the performance of real networks, since in many communication systems the strongest overlapping packet may *capture* the receiver and thus be correctly received. By taking advantage of the capture effect, it is possible to improve network performance.

The capture effect has been investigated in [8]-[10] for the classic slotted ALOHA random access scheme, under the unrealistic hypothesis that the combined traffic of new and retransmitted packets forms a stationary and independent Poisson process.

In this paper we investigate the effect of capture on collision-resolution algorithms. We focus here on the basic static binary tree CRA (having maximal throughput of 0.346) due to Capetanakis [1] and Tsybakov and Mikhailov [2], which is known to be very flexible and insensitive to channel errors [3] (the ideas and modifications presented in this paper can be implemented in other tree CRA's as well). In the present algorithm, the conventional three channel states (idle, success, collision) are enhanced by a fourth state, *capture*, which occurs when one of several transmitted packets is received successfully.

The paper is organized as follows. In Section II we describe the exact model, its basic assumptions, and the two possible feedback information channels according to whether or not the receiver can distinguish between success and capture. Two models are described: the general model for which the extended tree algorithms are described, and the simplified model (which is a special case of the general model) which is used in the analysis part. In Sections III and IV we show how the basic tree CRA [1], [2] can be applied to the situations where the receiver can distinguish between success slots and capture slots and to the one where it cannot, respectively. In each case; explicit recursive equations for the simplified model are given from which the average collision-resolution interval (CRI) lengths, conditioned on the number and type of packets involved in the initial collision, can be obtained. In addition, we obtain the maximal throughput in each case. Finally, we show that in presence of capture the performance of the network improves, i.e., the maximal throughput increases.

Extensions of the simplified model which can be analyzed using the same techniques are given in Section V, where we also show that with these extensions a packet-radio satellite channel can be modeled. Ground packet-radio networks are also considered in this section. In these networks, capture of a packet depends on the distances between the transmitting nodes from the common receiver and their transmission power. This situation seems analytically intractable, so we give only simulation results for this case. In the final section we summarize our results and discuss briefly how the capture effect affects the modified CRA [2], [3].

II. The Model

We consider accessing a common receiver by many nodes. The forward channel (from the nodes to the receiver) is assumed to be a noiseless, time-slotted, collision-type common radio channel. Each node can transmit one packet at a time whose duration is exactly one slot. In the general model we assume that the nodes of the network are divided into a finite number of groups, each containing any number (one, finite, or infinite) of nodes. This division is done so that a capture never occurs within the nodes of the same group. The groups are uniquely (yet arbitrarily) identified by a group identity number. Each transmitted packet carries the group identity number of its originator node.

The possible events that may occur during each slot are as follows.

a) Idle slot—no node is transmitting during the slot, in which case the slot is idle.

b) Success slot—exactly one node uses the channel, in which case its packet is successfully received.

c) Collision slot—two or more nodes use the channel but none of the individual transmitted packets can be reconstructed at the receiver. All packets must be retransmitted at some later time.

d) Capture slot—two or more nodes use the channel as in c), but in this case we assume that one of the packets captures the channel and, therefore, is successfully received at the receiver. All other packets cannot be reconstructed at the receiver and must be retransmitted at some later time. The collection of nodes which must retransmit their packets after a capture slot forms a *capture set*. Clearly, as said before, packets from the capture set carry a different group identity

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The authors are with the Department of Electrical Engineering, Technion—Israel Institute of Technology, Haifa, 32000, Israel.

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number than the successfully received packet. (No capture is allowed within the nodes of the same group.)

The feedback channel from the common receiver is assumed to be a noiseless broadcast channel. Through it the receiver informs all nodes, immediately at the end of each slot, of what happened during that slot. If any of the above events a), b), or c) occur, then the receiver broadcasts LACK, ACK, or NACK, respectively. (It is shown later that in some cases there is no need to distinguish between LACK and ACK.) If the above event d) occurs, we distinguish between two possible feedback information channels. The first is called feedback with capture (FWC). With FWC the receiver is able to distinguish between capture and success slots, i.e., it is able to detect that while it has received a packet, at least one more packet was simultaneously transmitted. In this case the receiver broadcasts CAPT to all nodes whenever d) occurs. This CAPT message carries the group identity number of the successfully received packet. The second feedback information channel is called feedback without capture (FWOC). With FWOC the receiver cannot distinguish capture from success slots, i.e., it cannot decide whether a correctly received packet was the only transmitted packet. In this case the receiver broadcasts ACK and the group identity number of the successfully received packet to all nodes in the network. In all cases, we assume that the propagation delay is negligible, so the feedback information of a certain slot can be used to determine who should transmit in the next slot.

We are interested in collision resolution algorithms that correspond to the above feedback information channels. By a CRA we mean a protocol for the transmission and retransmission of packets by the individual nodes, with the property that after an initial collision or a capture, all packets involved are eventually retransmitted successfully and all nodes, not only those who transmitted in the initial slot, eventually and simultaneously become aware that these packets have been successfully retransmitted [3]. We say that a collision (or capture) is resolved precisely when all nodes become aware that all the colliding packets have been successfully retransmitted. The time elapsed from an initial collision (capture) until it is resolved is called the collision-resolution interval (CRI).

We focus here on the basic CRA due to Capetanakis and show how it can be applied to the various feedback information channels. Specifically, we enumerate the actions an individual node involved in an initial collision (or capture) must take, and show how individual nodes determine the end of a CRI.

For the purpose of analyzing average CRI lengths and throughputs, we simplify the general model presented earlier. In this simplified model, the nodes of the network are divided into two priority groups—the dominating group (DG) and the nondominating group (NDG). Only packets transmitted by nodes from the DG can be captured at the receiver. We further assume that a capture slot occurs whenever exactly one node from the DG and one or more nodes from the NDG use the channel during that slot. For all other slots in which two or more nodes use the channel, the result is a collision slot. Notice that in the simplified model only two different group identity numbers are needed. Several extensions to the simplified model that can be analyzed using the techniques presented in this paper are discussed in Section V.

Two important measures of performance characterize a CRA. The first is the average collision-resolution interval length, conditioned on the number of packets of each group involved in the initial collision. This quantity is independent of the arrival process of new packets into the system or the conflict generation process. For the simplified model we shall derive recursive equations from which this traffic independent quantity can be obtained.

The second performance measure is the maximum attainable throughput, which is the maximum arrival rate of new packets into the system for which the system is stable. This quantity depends on the arrival process and the access scheme. We use numerical computation to obtain it for Poisson arrival processes.

III. CRA FOR FEEDBACK WITH CAPTURE (FWC)

In feedback with capture (FWC) where the receiver can distinguish between success slots and capture slots, it is possible to apply the tree CRA [1]–[3]. According to the tree CRA, after a collision, all nodes involved flip a binary coin; those flipping 0 retransmit in the very next slot, while those flipping 1 retransmit immediately after the collision (if any) among those flipping 0 has been resolved. No new packets may be transmitted until after the initial collision is resolved.

We assume that a capture slot is interpreted as such by *all* nodes in the network. This means that all packets belonging to a capture set must be retransmitted and, eventually, be correctly received at the common receiver before any other packets may be (re)transmitted.

Two possible actions might be taken by nodes from the capture set that are involved in a capture slot. One possibility is to perform the exact tree CRA described above, i.e., flip a coin, etc. We shall call this Scheme 1. The other possibility is to transmit during the slot immediately following the capture slot and then continue to perform the tree CRA. We call this Scheme 2. The motivation for Scheme 2 is the possibility that there is only a single node in the capture set of a particular slot, in which case it will take exactly one additional slot to resolve the collision. Examples of both schemes are depicted in Figs. 1 and 2, where an initial collision between four nodes A, B, C, D is resolved. These examples are for the general model, and thus, some of its actions cannot be observed in the simplified model.

Note that in both Schemes 1 and 2, the receiver need not distinguish between idle and success slots. This means that the LACK broadcast message is redundant and the receiver can broadcast ACK whenever a slot is either idle or successful.

Traffic-Independent Analysis of the CRA-FWC for the Simplified Model

Let a collision-resolution interval (CRI) be the interval from an initial collision until this collision is resolved. As stated before, we assume that no new packets may be transmitted until a CRI is completed. Let X be the number of packets transmitted in the first slot of a CRI and let $Y (Y \leqslant X)$ of these packets belong to the DG. Let Z denote the length (in slots) of the CRI. Here we are interested in the conditional mean CRI, $L_{n,k}$, defined as

$$L_{n,k} = E[Z/X = n, Y = k] \tag{1}$$

which is the average number of slots needed to resolve a collision between n nodes, of which k ($k \leqslant n$) belong to the DG (and hence, $n - k$ belong to the NDG).

In the following analysis we assume that nodes from the NDG perform the tree CRA after a capture slot (Scheme 1). At the end of this subsection we shall remark on the other possibility that they retransmit during the slot just after the capture slot and then continue to perform the tree CRA (Scheme 2).

It is easy to realize that $L_{0,0} = L_{1,0} = L_{1,1} = 1$. When $n \geqslant 2$, $k = 0$, there is a collision between n nodes that all belong to the NDG. Let p_1 be the probability of flipping 0 by a node from the NDG; then the probability that j of these colliding nodes flip 0 is just

$$P_n^1(j) = \binom{n}{j} p_1^j (1 - p_1)^{n-j} \tag{2}$$

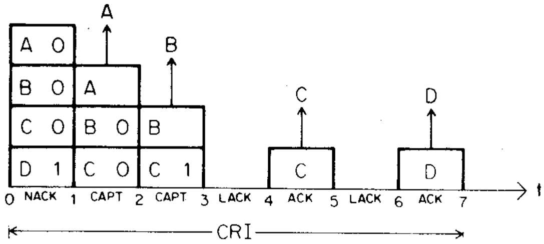

Fig. 1. Example for FWC—Scheme 1: A 0—node A transmits and flips 0.

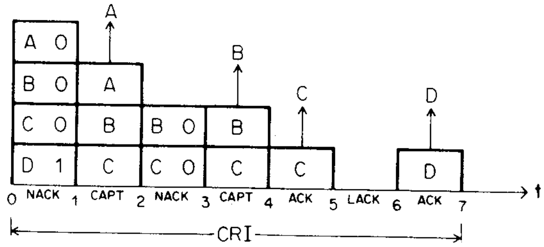

Fig. 2. Example for FWC—Scheme 2.

in which case

$$L_{n,0} = 1 + \sum_{j=0}^{n} P_n{}^1(j)[L_{j,0} + L_{n-j,0}] \tag{3}$$

from which we obtain that for $n \geqslant 2$

$$L_{n,0} = \frac{1 + \sum_{j=0}^{n-1} P_n{}^1(j)L_{j,0} + \sum_{j=1}^{n} P_n{}^1(j)L_{n-j,0}}{1 - P_n{}^1(n) - P_n{}^1(0)}. \tag{4}$$

Now if $n = 2$ and $k = 1$, then exactly one node from each of the DG and NDG transmits, a capture occurs, and the "collision" will be resolved after exactly two additional slots, that is,

$$L_{2,1} = 3. \tag{5}$$

When $n \geqslant 3$ and $k = 1$, one node from the DG and $n - 1$ nodes from the NDG transmit at the same time. Again a capture occurs at the first slot of the CRI, and still a collision between $n - 1$ remaining nodes that belong to the NDG has to be resolved. Since the collision occurs in the first slot, we have

$$L_{n,1} = L_{n-1,0} \qquad n \geqslant 3. \tag{6}$$

Finally, for $2 \leqslant k \leqslant n$, n nodes collide in the first slot, k of them from the DG and $n - k$ of them from the NDG. Let p_0 be the probability of flipping 0 by a node from the DG. Then the probability that i of k colliding nodes flip 0 is

$$P_k{}^0(i) = \binom{k}{i} p_0{}^i(1 - p_0)^{k-i}. \tag{7}$$

The probability that exactly i of k nodes from the DG and j of $n - k$ nodes from the NDG will flip 0 is just

$$Q_{n,k}(j, i) = P_{n-k}{}^1(j)P_k{}^0(i) \tag{8}$$

in which case the expected CRI length is given by

$$L_{n,k} = 1 + \sum_{i=0}^{k} \sum_{j=0}^{n-k} Q_{n,k}(j, i)[L_{i+j,i} + L_{n-i-j,k-i}] \tag{9}$$

from which we obtain that for $2 \leqslant k \leqslant n$,

$$L_{n,k} = \frac{1 + \sum\limits_{\substack{i=0 \\ (i,j)\neq(0,0)}}^{k} \sum\limits_{j=0}^{n-k} Q_{n,k}(j, i)L_{i+j,i} + \sum\limits_{\substack{i=0 \\ (i,j)\neq(k,n-k)}}^{k} \sum\limits_{j=0}^{n-k} Q_{n,k}(j, i)L_{n-i-j,k-i}}{1 - Q_{n,k}(0, 0) - Q_{n,k}(n - k, k)}. \tag{10}$$

As we see, $L_{n,k}$ is determined recursively from (2)–(10) using the initial conditions $L_{0,0} = L_{1,0} = L_{1,1} = 1$.

Regarding Scheme 2, we notice that the same equations (2)–(10) can be used to determine $L_{n,k}$ in this case, except that (5) and (6) are replaced by the following equations:

$$L_{2,1} = 2 \tag{11}$$

and

$$L_{n,1} = 1 + L_{n-1,0} \qquad n \geqslant 3. \tag{12}$$

The reason for this difference is that when exactly one node from the NDG is involved in a capture, then it will successfully transmit after the capture slot; therefore (11) holds. If more than one node from the NDG is involved in a capture, the slot just after the capture slot will be a collision slot, hence (12).

In Table I we give $L_{n,k}$ for FWC for different values of n and k, as derived from the recursive equations when $p_0 = p_1 = 0.5$.The lower line in each row of the table corresponds to Scheme 1, while the upper line corresponds to Scheme 2. As expected, the latter action outperforms the former when the number of nodes from the DG is large compared to the total number of nodes. When the number of nodes from the DG is small compared to the total number of nodes, it is better to use Scheme 1, where the tree is restarted at the root immediately after a capture slot. This behavior is also demonstrated in Fig. 3, where the quantity $n/L_{n,k}$ is plotted as a function of the number of nodes from the DG for $n = 24$. Note that the average CRI length for the basic tree algorithm without capture is just $L_{n,0}$ or $L_{n,n}$.

Maximal Throughput for Poisson Arrival Process

Let the packet generation process at DG and NDG nodes be Poisson with rates λ_{DG} and λ_{NDG}, respectively. Let also $\lambda_T = \lambda_{DG} + \lambda_{NDG}$ and $\alpha = \lambda_{DG}/\lambda_T$. Then $L_n{}^{(\alpha)}$, the average CRI length given that n packets collided in the initial slot, is computed as

$$L_n{}^{(\alpha)} = \sum_{k=0}^{n} L_{n,k} \binom{n}{k} \alpha^k(1 - \alpha)^{n-k} \tag{13}$$

where $L_{n,k}$ values have been calculated in the previous section.

Knowing $L_n{}^{(\alpha)}$, the maximal throughput λ_T is obtained as follows. Let $L(i, \alpha)$ be the length of the ith CRI given α. The resulting Markov chain $\{L(i, \alpha)\}$ is ergodic whenever there exists N, $\epsilon > 0$ such that [11]

$$E[L(i + 1, \alpha)/L(i, \alpha) = j] < (1 - \epsilon)j \qquad \forall j > N. \tag{14}$$

TABLE I

$L_{n,k}$ FOR FWC. LOWER LINE IN EACH ROW—SCHEME 1. UPPER LINE IN EACH ROW—SCHEME 2.

k \ n	4	8	16	24	32
0 or n	10.52	22.09	45.17	68.25	91.33
$\frac{n}{4}$	8.67 7.67	18.73 17.97	38.86 37.63	58.97 57.27	79.09 76.90
$\frac{n}{2}$	7.76 8.24	17.38 17.77	36.39 37.03	55.37 56.27	74.35 75.50
$\frac{3n}{4}$	8.38 9.10	18.43 19.41	38.37 40.01	58.29 60.62	78.21 81.23

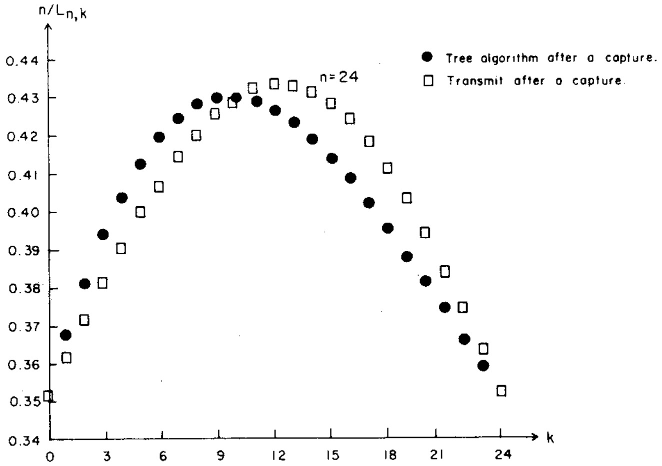

Fig. 3. $n/L_{n,k}$ versus number of nodes from the DG for FWC.

For Poisson arrival process, (14) becomes

$$\sum_{n=0}^{\infty} E[L(i+1, \alpha)/\text{number of arrivals in } i\text{th CRI} = n] \cdot \frac{(\lambda_T j)^n \exp\{-(\lambda_T j)\}}{n!}$$

$$= \sum_{n=0}^{\infty} L_n^{(\alpha)} \frac{(\lambda_T j)^n \{\exp - (\lambda_T j)\}}{n!} < (1-\epsilon) j \qquad \forall j > N. \tag{15}$$

One would then look for a linear bound on the expectation

$$L_n^{(\alpha)} < a(\alpha) \cdot n + b(\alpha) \tag{16}$$

so that the stability condition in (15) would transform into

$$\lambda_T < 1/a(\alpha). \tag{17}$$

The analytical evaluation of $a(\alpha)$ seems to be intractable, so numerical computation has been used, first calculating $L_n^{(\alpha)}$ from (13) and then checking the inquality (17) up to $n = 32$. The results are summarized in Table II. For $\alpha = 0$ or $\alpha = 1$,

TABLE II

MAXIMAL THROUGHPUTS FOR FWC. LOWER LINE IN EACH ROW—SCHEME 1. UPPER LINE IN EACH ROW—SCHEME 2.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$a(\alpha)$	2.72 2.65	2.53 2.51	2.47 2.42	2.40 2.40	2.38 2.41	2.39 2.46	2.45 2.53	2.55 2.63	2.70 2.75
λ_T^*	0.367 0.398	0.387 0.413	0.404 0.416	0.416 0.414	0.420 0.406	0.418 0.395	0.408 0.380	0.392 0.363	0.370 0.346

$a(\alpha) = 2.89$ and $\lambda_T^* = 0.346$ for both schemes, just as for the basic tree algorithm.

From Table II it is evident that when the ratio α is less than 0.4, Scheme 1 outperforms Scheme 2, while the reverse is true for $\alpha > 0.4$. When α is optimally chosen, the maximal attainable throughput increases up to 0.416 for Scheme 1 (20 percent increase compared with the case of no capture) and up to 0.420 for Scheme 2.

IV. CRA FOR FEEDBACK WITHOUT CAPTURE (FWOC)

We will refer back to the general model. In the FWOC the receiver cannot distinguish between capture and success slots. With FWOC, as with FWC, it is possible to extend the basic tree CRA as stated in Section III. The problem here is to determine what action should be taken by the nodes from the capture set that transmit a packet and receive ACK with a different group identity number. Notice that only these nodes from the capture set know that the receiver has captured a packet.

One possible scheme is called "wait for partial conflict resolution." According to this scheme, the conflict resolution interval that corresponds to a given initial collision is divided into parts called partial conflict resolution intervals (PCRI). The first part ends when all nodes involved in the initial collision, except those nodes from capture sets that were involved in a capture in the first part, have successfully transmitted their packets. Those nodes from capture sets that were involved in a capture wait until the first interval ends, and then retransmit their packets. The second part of the CRI is dedicated to resolve conflicts (if any) between these nodes. Nodes from capture sets of the second PCRI will retransmit their packets in the third PCRI and so on. The conflict is finally resolved when an empty PCRI is detected (a single idle slot).

Another possible scheme is called "send in the next slot." According to this scheme, whenever a node detects that it belongs to a capture set, it retransmits in the next slot. Here, as in the previous scheme, the CRI is composed of several parts. During every PCRI other than the first, only packets involved in a capture at the last slot of the previous PCRI are retransmitted. (If the last slot of a PCRI is idle, then there is no need for another PCRI and the collision is finally resolved.) An example of both schemes for the general case is depicted in Figs. 4 and 5.

Note that the above two schemes can be combined to provide other schemes in which each node from a capture set determines randomly (by flipping a coin, say) whether to act according to the "wait for partial conflict resolution" scheme or according to the "send in the next slot" scheme.

Traffic-Independent Analysis of the "Wait for Partial Conflict Resolution" Scheme for the Simplified Model

Let $P_n^1(i)$, $P_n^0(i)$, $Q_{n,k}(j, i)$ be defined as in Section III. In the simplified model, when the "wait for partial conflict resolution" scheme is used, the CRI consists of two parts. The first is the interval needed to partially resolve the initial collision, and the second is the interval needed to transmit packets from the NDG that delayed their retransmissions be-

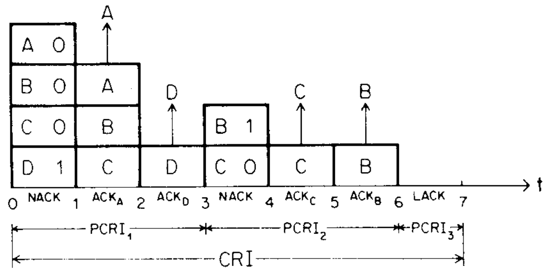

Fig. 4. Example for FWOC—"wait for partial conflict resolution."

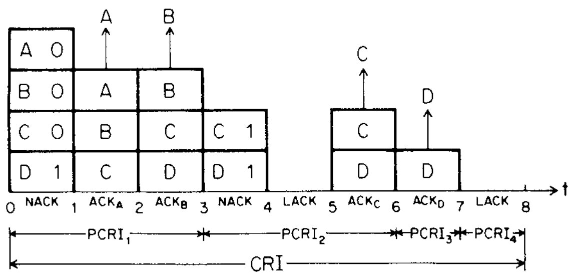

Fig. 5. Example for FWOC—"send in the next slot."

cause of capture (no capture occurs in this part). Let the lengths of these two intervals be denoted by Z and W, respectively. As in (1) we define the following conditional means:

$$L_{n,k} = E[Z/X = n, Y = k] \tag{18a}$$

$$\tilde{L}_{n,k} = E[W/X = n, Y = k]. \tag{18b}$$

Clearly, the average number of slots needed to resolve a collision between n nodes, of which k belong to the DG, is just $L_{n,k}{}^T = L_{n,k} + \tilde{L}_{n,k}$. It is easy to see that $L_{n,k}$ evolves according to (2)-(10) with the same initial conditions except that (5) and (6) should be replaced by

$$L_{n,1} = 1 \qquad n \geqslant 1. \tag{19}$$

The reason is that when exactly one node from the DG transmits at the first slot of a CRI, its packet is always captured, and therefore the initial "collision" is partially resolved.

We now show how $\tilde{L}_{n,k}$ is calculated recursively. Let $P_{n,k}(l)$, $0 \leqslant l \leqslant n - k$, be the conditional probability that exactly l nodes from the NDG have been involved in capture slots during the first part of the CRI, given that n nodes, of which k are from the DG, transmit in the first slot of a CRI. We shall first show how these probabilities are calculated. The following relations clearly hold:

$$P_{n,n}(0) = 1 \qquad n \geqslant 0;$$

$$P_{n,n}(l) = 0 \qquad n \geqslant 0, 1 \leqslant l \leqslant n \tag{20a}$$

$$P_{n,0}(0) = 1 \qquad n \geqslant 1;$$

$$P_{n,0}(l) = 0 \qquad n \geqslant 1, 1 \leqslant l \leqslant n \tag{20b}$$

$$P_{n,1}(n-1) = 1 \qquad n \geqslant 1;$$

$$P_{n,1}(l) = 0 \qquad n \geqslant 1, 0 \leqslant l \leqslant n - 2 \text{ or } l = n. \tag{20c}$$

Relation (20a) holds since for $k = n$, there are no nodes from the NDG and therefore, with probability 1, $l = 0$. Relation (20b) holds since if no node from the DG transmits in the first slot of a CRI, then no capture will occur during this CRI and therefore, with probability 1, $l = 0$. Finally, (20c) holds since in the case where exactly one node from the DG transmits in the first slot of a CRI, all the other $n - 1$ nodes transmitting during this slot receive ACK and therefore retransmit again at the end of the first part of the CRI.

Let us now determine $P_{n,k}(l)$ for $n > k \geqslant 2$ and $0 \leqslant l \leqslant n - k$. In this case, the probability that exactly i of the nodes from the DG and j of the nodes from the NDG will flip 0 is $Q_{n,k}(j, i)$ defined in (8). Consequently,

$$P_{n,k}(l) = \sum_{(i,j,m) \in S} Q_{n,k}(j, i) P_{i+j,i}(m) P_{n-i-j,k-i}(l - m) \tag{21a}$$

where

$$S = \{(i, j, m) : 0 \leqslant i \leqslant k, \quad 0 \leqslant j \leqslant n - k, \quad 0 \leqslant m \leqslant l\}. \tag{21b}$$

From (21) using (20a), we obtain

$$P_{n,k}(l) = \frac{\sum_{(i,j,m) \in S'} Q_{n,k}(j, i) P_{i+j,i}(m) P_{n-i-j,k-i}(l - m)}{1 - Q_{n,k}(0, 0) - Q_{n,k}(n - k, k)} \tag{22a}$$

where

$$S' = S - \{k, n - k, l\} - \{0, 0, 0\}. \tag{22b}$$

Clearly, $P_{n,k}(l)$ can be determined recursively through (22) using the initial conditions (20).

Finally, $\tilde{L}_{n,k}$ for $n > k \geqslant 1$ is calculated via

$$\tilde{L}_{n,k} \sum_{l=0}^{n-k} P_{n,k}(l) L_{l,0} \tag{23}$$

since the conflict between l nodes from the NDG is resolved on the average in $L_{l,0}$ slots. Also $\tilde{L}_{n,n} = \tilde{L}_{n,0} = 1$ since when n nodes, all from the same group, transmit at an initial slot, then a capture will not occur.

Traffic-Independent Analysis of the "Send in the Next Slot" Scheme for the Simplified Model

As described above, when the "send in the next slot" scheme is used, the CRI consists of two parts. During the first part, all nodes from the DG and some (possibly all) nodes from the NDG successfully transmit their packets. The second part is devoted to retransmissions of packets (if any) from the NDG that were involved in a capture during the last slot of the first part. Let us call these packets *residual packets*. As will be clear from the following, a crucial quantity in the analysis of this scheme is the conditional probability distribution of the number of residual packets, given that the initial collision is between n nodes of which k are from the DG. Let us denote this probability by $P_{n,k}(l)$, i.e.,

$$P_{n,k}(l) = \text{Prob}\left\{ l \text{ residual packets} \middle/ \begin{array}{l} n \text{ nodes of which } k \text{ are} \\ \text{from the DG transmit at the} \\ \text{first slot of a CRI} \end{array} \right\}.$$

As in (20) it is easy to see that

$$P_{n,n}(0) = 1 \qquad n \geqslant 0;$$

$$P_{n,n}(l) = 0 \qquad n \geqslant 0, 1 \leqslant l \leqslant n \tag{24a}$$

$$P_{n,0}(0) = 1 \qquad n \geqslant 1;$$

$$P_{n,0}(l)=0 \qquad n\geqslant 1,\ 1\leqslant l\leqslant n \tag{24b}$$

$$P_{n,1}(n-1)=1 \qquad n\geqslant 1;$$

$$P_{n,1}(l)=0 \qquad n\geqslant 1,\ 0\leqslant l\leqslant n-2 \text{ or } l=n. \tag{24c}$$

Let us now determine $P_{n,k}(l)$ for $n>k\geqslant 2$ and $0\leqslant l\leqslant n-k$. The initial slot of the CRI contains n packets of which k are from the DG. With probability $Q_{n,k}(j,i)$, i of the nodes from the DG and j of the nodes from the NDG flip zero. Now a CRI of these $i+j$ nodes begins. With probability $P_{i+j,i}(m)$, $0\leqslant m\leqslant j$, the number of residual packets of this CRI is exactly m. Therefore, with probability $P_{i+j,i}(m)$ the first slot of the following CRI will contain $n-i-j+m$ nodes, $k-i$ of them from the DG. With probability $P_{n-i-j+m,k-i}(l)$ we shall have l residual packets at the end of the CRI. Consequently, for $0\leqslant l\leqslant n-k$

$$P_{n,k}(l)=\sum_{(i,j,m)\in S} Q_{n,k}(j,i)P_{i+j,i}(m)P_{n-i-j+m,k-i}(l) \tag{25a}$$

where

$$S=\{(i,j,m): 0\leqslant i\leqslant k,\quad 0\leqslant j\leqslant n-k,\quad 0\leqslant m\leqslant j\}. \tag{25b}$$

From (25) and (24) we obtain for $1\leqslant l\leqslant n-k$:

$$P_{n,k}(l)=\frac{\displaystyle\sum_{(i,j,m)\in S'} Q_{n,k}(j,i)P_{i+j,i}(m)P_{n-i-j+m,k-i}(l)}{1-Q_{n,k}(0,0)} \tag{26a}$$

where

$$S'=S-\{(0,0,0)\}-\{k,n-k,l\} \tag{26b}$$

and

$$P_{n,k}(0)=\frac{\displaystyle\sum_{(i,j,m)\in S''} Q_{n,k}(j,i)P_{i+j,i}(m)P_{n-i-j,k-i}(0)+Q_{n,k}(n-k,k)}{1-Q_{n,k}(0,0)} \tag{27a}$$

where

$$S''=S-\{(i,j)=(0,0)\}-\{(i,j)=(k,n-k)\}. \tag{27b}$$

From (26) and (27) it is clear that $P_{n,k}(l)$ can be determined recursively.

Now that we have determined the conditional probabilities $P_{n,k}(l)$, we are ready to calculate the conditional mean length of a CRI. Let the first slot of a CRI contain n packets, k of which belong to the DG. Let $L_{n,k}$ and $\tilde{L}_{n,k}$ be the mean lengths of the first and the second parts of this CRI, respectively. It is easy to see that

$$\tilde{L}_{n,k}=\sum_{m=0}^{n-k} P_{n,k}(m)L_{m,0}. \tag{28}$$

The reason is that the second part of the CRI is devoted exclusively to retransmissions of the residual packets from the NDG which have a distribution $P_{n,k}(m)$.

It is also easy to realize that $L_{n,0}$, $n\geqslant 2$, evolves as in (4) with the initial conditions $L_{1,0}=L_{0,0}=1$. Also it is clear that

$$L_{n,1}=1 \qquad n\geqslant 1 \tag{29}$$

since when exactly one node from the DG transmits in the first slot of the CRI, its packet is always captured, and therefore, the first part of the CRI immediately ends. For $n>k\geqslant 2$ we have

$$L_{n,k}=1+\sum_{i=0}^{k}\sum_{j=0}^{n-k} Q_{n,k}(j,i)\cdot\left[L_{i+j,i}+\sum_{m=0}^{j} P_{i+j,i}(m)L_{n-i-j+m,k-i}\right]. \tag{30}$$

From (30) using (24) we obtain

$$L_{n,k}=\frac{\displaystyle 1+\sum_{(i,j)\in S} Q_{n,k}(j,i)L_{i+j,i}+\sum_{(i,j,m)\in S'} Q_{n,k}(j,i)P_{i+j,i}(m)L_{n-i-j+m,k-i}}{1-Q_{n,k}(n-k,k)-Q_{n,k}(0,0)} \tag{31a}$$

where

$$S=\{(i,j): 0\leqslant i\leqslant k,\quad 0\leqslant j\leqslant n-k,\quad (i,j)\neq(k,n-k)\} \tag{31b}$$

$$S'=\{(i,j,m): 0\leqslant i\leqslant k,\quad 0\leqslant j\leqslant n-k,\quad 0\leqslant m\leqslant j,\quad (i,j,m)\neq(0,0,0)\}. \tag{31c}$$

Therefore, $L_{n,k}$ is determined recursively through (31). Finally, $L_{n,n}=L_{n,0}$ for $n\geqslant 0$.

We have shown how to calculate the mean conditional lengths of the two parts of the CRI, and the mean conditional total length of the CRI is just $L_{n,k}{}^T=L_{n,k}+\tilde{L}_{n,k}$.

In Table III we give $L_{n,k}{}^T$ for FWOC for different values of n and k, as derived from the recursive equations when $p_0=p_1=0.5$. The upper line in each row of the table corresponds to the "wait for partial conflict resolution" scheme, while the lower line corresponds to the "send in the next slot" scheme. As expected, the "send in the next slot" scheme is better than the "wait for partial conflict resolution" scheme for most situations. The reason is that the former scheme uses more efficiently the slots unoccupied by packets from the DG, and it also favors resolution of several small population conflicts of packets from capture sets, as opposed to resolution of one large conflict preferred in the latter scheme. This behavior is also demonstrated in Fig. 6, where the quantity $n/L_{n,k}{}^T$ is plotted as a function of the number of nodes from the DG for $n=24$. Note that here the CRI length of the basic tree algorithm is just $L_{n,0}{}^T-1$ or $L_{n,n}{}^T-1$.

Maximal Throughput for Poisson Arrival Process

Using the same techniques of Section III [see (13)-(17)], we calculated the maximal throughput for Poisson arrival processes for both schemes. The results are shown in Table IV.

TABLE III
$L_{n,k}{}^T$ FOR FWOC. LOWER LINE IN EACH ROW—SEND IN THE NEXT SLOT. UPPER LINE IN EACH ROW—WAIT FOR PARTIAL CONFLICT RESOLUTION.

k \ n	4	8	16	24	32
0 or n	11.52	23.09	46.17	69.25	92.33
$\frac{n}{4}$	9.67 9.67	19.73 20.23	41.39 41.64	63.17 63.16	84.96 84.69
$\frac{n}{2}$	8.68 9.18	19.20 19.20	40.56 39.77	61.95 60.26	83.35 80.75
$\frac{3n}{4}$	8.67 8.67	19.78 19.46	41.54 40.00	63.38 60.49	85.23 80.95

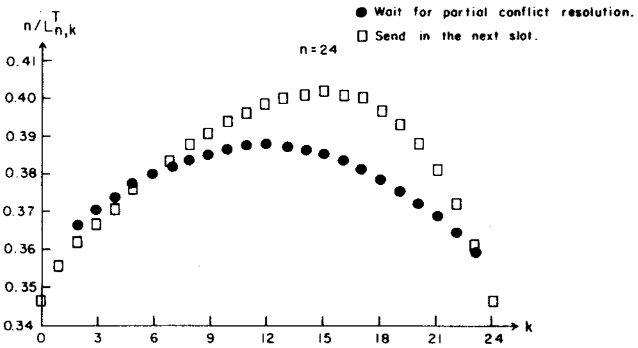

Fig. 6. $n/L_{n,k}{}^T$ versus number of nodes from the DG for FWOC.

TABLE IV
MAXIMAL THROUGHPUTS FOR FWOC. LOWER LINE IN EACH ROW—SEND IN THE NEXT SLOT. UPPER LINE IN EACH ROW—WAIT FOR PARTIAL CONFLICT RESOLUTION.

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$a(\alpha)$	2.80 2.80	2.75 2.72	2.70 2.66	2.68 2.60	2.67 2.56	2.68 2.54	2.71 2.54	2.75 2.59	2.81 2.68
λ_T^*	0.357 0.357	0.364 0.367	0.370 0.376	0.373 0.384	0.374 0.390	0.373 0.394	0.369 0.393	0.363 0.387	0.356 0.373

For $\alpha = 0$ or $\alpha = 1$, $a(\alpha) = 2.89$ and $\lambda_T^* = 0.346$ for both schemes, just as for the basic tree algorithm.

As is apparent from Table IV, when the ratio α is greater than 0.1, the "send in the next slot" scheme outperforms the "wait for partial conflict resolution" scheme. When α is optimally chosen, the maximal attainable throughput increases up to 0.394 for the former (13 percent increase compared to the case of no capture) and up to 0.374 for the latter.

V. Extensions to the Simplified Model

In this section we discuss two possible extensions to the simplified model considered in Section II. The first possible extension is to divide the nodes of the network into M priority groups instead of only two groups. We refer to this model as an *M-discrete model*. Let the M groups of nodes be denoted by $A_1, A_2, \cdots, A_M$. In the M-discrete model we say that group A_i *dominates* group A_j whenever $i > j$. The domination has the following sense: if a single packet originated at A_i and an arbitrary number of packets originated at groups dominated by A_i are transmitted during the same slot, then the common receiver will capture the packet which originated at A_i.

In previous sections we have analyzed the *2-discrete* model. Collision resolution algorithms for the M-discrete model are similar to the algorithms presented in Sections III and IV. We may note here that for proper operation of these algorithms, there is a need for each group to have its own identity. Methods similar to those used in the previous sections can be used to carry out the analysis when $M > 2$. Clearly, the dimensionality of the problem increases linearly in M, but no new ideas should be introduced, and in order to save space, we omit the details here.

Another extension to be considered is the case that a node from the DG can be captured at the receiver only if fewer than K nodes from the NDG are transmitting at the same time. This case can be easily analyzed using the same approach as in Sections III and IV. For example, for FWC Scheme 1, the only change that is needed is to use (6) only for $n < K$.

The M-discrete model with or without the latter extension can be used to model a packet-radio satellite channel with M groups of ground stations, where each group differs from others by the transmission powers of the stations. Within this environment we can assume that the dominant effect is the transmission power rather than the station-satellite distance. This is not the case in a ground packet-radio network, where both power and distance must be considered.

For this purpose we suggest another possible extension that will be called a *continuous model*. According to this model, the transmitters have different transmission powers and different distances from the common receiver. Let P_i and R_i be the power and distance of the ith transmitter, respectively, and let us assume that n transmitters, i_1 to i_n, transmit simultaneously during some slot. Then the packet originated at i_1 will be captured if

$$T \frac{P_{i_1}}{R_{i_1}{}^2} > \sum_{j=2}^{n} \frac{P_{i_j}}{R_{i_j}{}^2} \tag{32}$$

where T is a capture factor that can take values from [0, 1]. (Clearly, if $T = 0$, capture never occurs, while if $T = 1$ and exactly two nodes transmit, then a capture will definitely occur.) $P_j/R_j{}^2$ is clearly proportional to the jth transmitter power at the common receiver antenna. It should be noted that in the continuous model, each node has a different group identity number which can be considered here as a node identity. The CRI in this case is divided into an *a priori* unknown number of partial collision-resolution intervals, and the conflict is finally solved when the nodes detect an empty partial CRI. (If the chosen algorithm is "send in the next slot," then it is sufficient to detect that the last slot of such a CRI is empty.)

The continuous model is difficult to analyze and we shall present several simulation results regarding this model. Three parameters influence the throughput of the network with this model:

i) the capture factor—T
ii) the topology of the network
iii) the transmission powers of the nodes.

In our simulations the transmitters are uniformly distributed within a circle of one unit radius around the receiver, and have the same transmission power. The capture factor ranges between 0 and 1. The throughput for networks having 20 and 50 nodes is shown in Fig. 7. As expected, we see that the throughput increases as the capture factor increases.

VI. Summary and Discussion

In this paper we have investigated the effect of capture on the basic tree collision-resolution algorithm [1]–[3]. We

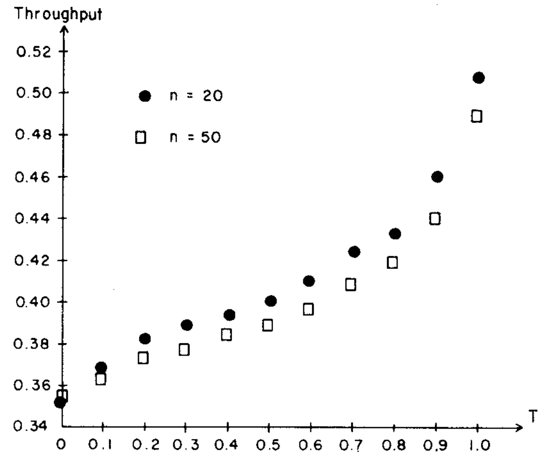

Fig. 7. Throughput versus capture factor: simulation results.

have considered both the situations where the receiver can distinguish between success and capture and where it cannot. For each of these cases we described collision-resolution algorithms and derived explicit recursive equations from which the performance of these algorithms can be evaluated. As expected, the performance of the network improves when packets might be captured.

The idea introduced by Massey [3] and independently by Tsybakov and Mikhailov [2], of saving slots which are known to contain collisions, can be easily incorporated into our algorithms to further improve the performance. For example, for FWC Scheme 2, and for optimal division of rates between groups, the maximal attainable throughput increases up to 0.445 (compared to 0.420 without using this idea). However, it should be remembered that this improved algorithm is very sensitive to channel and feedback errors [3] (it can lead to a deadlock).

A final remark should be included regarding the biases p_0 and p_1 of the coins, which nodes use to traverse their respective trees. In all our numerical examples we set $p_0 = p_1 = 0.5$. Obviously, one can optimize the bias values in order to improve further the performance of the network. This can be done by numerical search using the recursive equations developed in this paper.

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Israel Cidon was born in Israel in 1954. He received the B.Sc. (summa cum laude) and D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, in 1980 and 1984, respectively, both in electrical engineering.

From 1977 to 1980 he was a Consulting R&D Engineer involved in the design of various microprocessor-based equipment. From 1980 to 1983 he was a Teaching Assistant and a Teaching Instructor at the Technion. In 1984 he joined the Faculty of the Department of Electrical Engineering at the Technion, where he is currently a Lecturer. His current research interests are in distributed algorithms and computer-communication networks.

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Moshe Sidi (S'77–M'82) was born in Israel in 1953. He received the B.Sc., M.Sc., and D.Sc. degrees from the Technion—Israel Institute of Technology, Haifa, in 1975, 1979, and 1982, respectively, all in electrical engineering.

From 1975 to 1981 he was a Teaching Assistant and a Teaching Instructor at the Technion in communication and data networks courses. In 1982 he joined the Faculty of the Department of Electrical Engineering at the Technion, where he is currently a Lecturer. During the academic year 1983–1984 he was a Postdoctoral Associate at the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology, Cambridge. His current research interests are in queueing theory and in the area of computer-communication networks.

Dr. Sidi received the 1981–1982 IEEE Communications Society Scholarship Award, the 1982 Landau Award for Research in the Field of Electrical Engineering, and the Rothschild Fellowship for 1983–1984.