

Management of Large-scale Multimedia Conferencing

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Abstract

The goal of this work is to explore management strategies and algorithms for large-scale multimedia conferencing over a communication network. Since the use of multimedia conferencing is still limited, the management of such systems has not yet been studied in depth. A well organized and human friendly multimedia conference management should utilize efficiently and fairly its limited resources as well as take into account the requirements of the conference participants. The ability of the management to enforce fair policies and to quickly take into account the participants preferences may even lead to a conference environment that is more pleasant and more effective than a similar face-to-face meeting. We suggest several principles for defining and solving resource sharing problems in this context. The conference resources which are addressed in this paper are the bandwidth (conference network capacity), time (participants' scheduling) and limitations of audio and visual equipment. The participants' requirements for these resources are defined and translated in terms of Quality of Service (QoS) requirements and the fairness criteria.

A suggested solution for the problem of Capacity Resource Management (CRM) allocation is the Extended Max Min Fairness (EMMF) criterion, an extension of the well-known Max Min Fairness criterion. Both centralized and distributed algorithms that satisfy this criterion are presented. Further trade-offs between fairness and total throughput are also suggested. The conference time allocation problem is defined and mapped to known problems of time scheduling which are widely discussed in the literature. We examine the well-known Generalized Processor Sharing system (GPS) in that context, and select (after some adaptation) its Worst-case Fair Weighted Fair Queuing (WF²Q) version, a scheduling policy based on the GPS system which satisfies the participants' requirements. Finally, we describe methods for combining the Time Resource Management (TRM) and the Capacity Resource Management (CRM) into a complete management of multimedia conferencing.

1 Introduction

The progress in computer networks technology accelerates the use of multimedia conferences that allow multi-level communication and collaboration between remote participants. The multimedia conference offers many advantages to the participants. It saves time and travel, avoids duplicate meetings and enables a high level of collaboration.

Video conferencing over a communication network consists of multiple participants equipped with various presentation tools communicating with each other utilizing the network services. Each participant operates his networked multimedia workstation independently of other participants. A participant may transmit to other participants a variety of information types such as real-time video streams, video and audio clips, slides and real-time shared white-board. The video conference participants act in a similar way as in a real conference room. They may wish to listen and watch a remote speaker, a multimedia presentation, or they may wish to speak or even interrupt a speaker.

When the number of participants is small, and the participants are not greedy, video conference management is not essential. The network capacity can be allocated in advance and kept fixed during the conference. Time management can be done by one of the participants functioning as a moderator, or can be self managed using the participants good behavior and ethics. However, when the conference becomes bigger, and its participants are greedy, it must be managed by the conferencing system to guarantee a level of fairness. Otherwise, the conference will collapse in terms of its usability and perceived value. For example, a lecture with a large number of participants all wanting to ask questions simultaneously. The lecturer's screen can not present all of the participants' figures simultaneously and the lecturer can not listen to all of them at the same time. Moreover, the ability to screen and control participants as well as providing fairness and take into account the participant preferences directly, can make the video conference into a collaboration environment which is much better than a large face-to-face meeting. Management of large scale video conferencing can provide essential qualities that can not be provided in a conference

room. The conference central management can prevent collisions, When multiple participants start speaking simultaneously or are prevented from speaking because of other participants interruptions. The presence of a conference management can guarantee the participants minimal sharing requirements. Each participant can define a set of preferences such as his minimum speaking duration during the conference and the favorite presenters (or presentations) he would like to follow. Each participant's preferences are handled privately by the conference management. The participants can be preallocated guaranteed conference resources and can also interactively present new preferences during the conference. The system can take these preferences into account for its online management. For example, a favorite participant may receive a longer speaking duration than others. The conference management can secure privacy that can not be provided in a conference room. For example, each participant can specify the participants who are allowed (or not allowed) to watch him and he can hold private discussions during a presentation without disturbing others.

The conference management should support the participants' demands by allocating its common resources to participants and other information sources. This allocation should be feasible, fair and humanly reasonable. The multi-party video conferencing management system faces conflict in two main system resources, conference time and network capacity. These resources are limited by the network's restrictions and by the participants' demands. Previous related research in networked multimedia such as [1] focuses on floor control, i.e., how to allow participants of networked multimedia applications to share remote devices. The aim of floor control is to guarantee mutually exclusive resource usage. For example, how to share a video stream. This paper examines a specific floor control problem of conference management, how to allow remote participants to collaborate and share fairly, efficiently and with no collisions the network common resources (time and capacity).

The conference duration is a limited resource. The conference might have a planned termination time and each participant may have his own time limits. The conference Time Resource Management (TRM) function is to allocate the time resource between participants according to their demands, i.e., to determine the par-

ticipants' transmission duration and order. The TRM can schedule more than one participant, so that a number of participants can transmit simultaneously to the conference. The TRM may control the participants' speaking volume. The current speaker is heard loudly, and the others at a lower volume.

The conference capacity is also a limited resource. The network capacity which is available to this conference on each network link is bounded. The Capacity Resource Management (CRM) function is to allocate capacity among all the multiple point-to-point and multicast transmissions. Capacity allocation affects the amount of concurrent sources that can be allocated in parallel, as well as video and voice quality. These limitations on resources lead to necessary management trade-offs, in the allocation of time and capacity among all the participants.

The conference management can be partitioned into two main management parts the TRM and the CRM. The TRM allocates time according to the source participants demands (can be related to speaker broadcast requests). The CRM allocates capacity according to the destination participants demands (can be related to destination viewing requests).

Each resource management part is made up of two components: the admission control management and the interactive management. The admission control management checks the feasibility of the participants' requirements at the beginning of the conference and checks the online requirements of participants wishing to join during the conference. The interactive management allocates resources according to the participants' interactive and a priori demands.

In order to allocate resources relative to the participants' needs, each participant defines his basic service demands. These demands are based on human concepts and should be expressed in terms of Quality of Service (QoS) constraints. The use of QoS terms to define the multimedia users needs was already mentioned in [2]. In this paper we focus specifically on multimedia conferencing, i.e., human participants taking part in a large scale video conference and wanting to guarantee their participation rights (speaking and watching) during the conference.

The conference management is also affected by the conference model. There are many natural conference models, such as the “lecture model” or the “peer meeting model”, that reflect meetings in daily life. We attempt to characterize several of these models and present simple examples for their implementation. A characterization of multimedia conferencing was already mentioned in [3]. This paper defines the methodologies of video conference management. It describes the concepts of management related to resources limitations and participants needs. It also suggests specific algorithms as examples of such concepts. There is still much research needed in order to reach a full video conference management that will satisfy participants needs and the various conference models.

In the next sections we discuss the following issues. Section 2 illustrates the main conference models, Section 3 defines the capacity allocation problem and extends the MMF allocation criterion to the Extended MMF (EMMF) criterion. Section 3.1 describes the central allocation algorithm which satisfies the EMMF criterion, and Section 3.2 describes the distributed implementation of EMMF. In order to increase the overall conference throughput, the EMMF fairness criterion is relaxed to the $\vec{\delta}$ EMMF criterion as described in Appendix A. The time allocation problem and the participants’ time requirements are defined in Sections 4.1 and 4.2 as a scheduling problem and are mapped to the known Generalized Processor Sharing system (GPS). This solution is extended to the Worst-case Fair Weighted Fair Queuing (WF²Q) policy which satisfies the formalized TRM problem. Section 5 describes simple integration examples of the TRM and the CRM into an entire management system.

2 The Conference Models

Large-scale video conferencing over a communication network is largely a future application. It is still unknown what the collection of dominant applications will be. For our research needs we informally suggest several conference models based on common day-to-day models. Later, we will discuss the influence of these models on the conference management architecture.

The Peer Meeting Model. The conference participants play a similar role in the conference. They may have a different degree of information to present, or different presentation capabilities. An example could be a business negotiation between several equal companies members or a standard committee meeting.

The Lecture Model. One of the conference participants is the lecturer, the rest are his audience. The lecturer is the conference speaker for most of the time and therefore employs most of its resources. The audience asks questions or makes brief remarks. They spend most of their time listening to the lecturer, or browsing through his supporting material.

The Parliament Model. Two types of request are available to a participant: Request for a speech and request for a remark. A speech is characterized by a long duration and low priority. Therefore, the system response time for such a request is relatively longer. A remark is a short speaking period with high priority, therefore, the system response time for such a request is relatively shorter. There is only one participant speaking at a time (another model can assume short interruptions to speakers). The number of speaking requests for each type is limited.

The Debate Model. There is a specially-designated group of participants that address the conference, all the others form the audience. The audience does not speak during the conference but interactively present their preferences as who they wish to see more. The participation order of those in the designated group and their speaking periods duration is determined interactively according to their rating. The designated participants are greedy, and spend all of the time allocated to them as, for example, in the election debate between several candidates competing for an office. Each candidate gets a period of time in which to explain and defend his opinions.

The Multi Group Model. This conference is characterized by working groups. A participant can be a member in more than one group. He may speak and listen only to his membership groups, i.e. when one of the participants is speaking he can be seen only by the groups that he chooses from his membership groups. An example would be a large meeting of different groups from different companies who want to consult one another during the conference, without interfering with other participants; they

also require the privacy of not being observed by their competitors.

The Dynamic Model. With this model participants can join or leave during the conference. The number of participants is not fixed. A participant who wishes to leave can do so without informing the conference management, but participants who wish to join the conference must receive approval from the conference management through an admission control policy. An example would be an Internet chat group.

3 Capacity Resource Management - (CRM)

The objective of Capacity Resource Management (CRM) is to allocate networking capacity among participants, conforming to the needs and constraints of the participants' requirements and the system capacity restrictions. Therefore, CRM should have information regarding the edge's free capacity, the information flow routing paths, and the participants' requirements. The CRM divides each edge free capacity between the specific information flows which pass through it. This division has multiple objectives that may conflict with each other. Such objectives may be maximizing the network total throughput, fair allocation, and satisfying the participants' requirements. It is assumed that participant information flows can be delivered in variable grades. For example, a real-time video can be allocated any amount of bandwidth resulting in a certain grade of quality. The following sections present the main requirements of the participants, formally define participants fairness, and suggest several algorithms for given network models and given CRM objectives.

Capacity Requirements

Participant capacity requirements are derived from several factors, such as the participant's applications, the participant's equipment limitations and the conference model. Video conference participants may specify their choice of sources, such as the list of sources they wish to watch and at which quality level. There are several ways in which such preferences can be presented. Destination may present to the CRM the ratio of its required allocation between the different sources. Destination may also determine, for each source, the maximum and minimum rate that it is

willing or able to receive. For example the minimum rate is related to the minimum video quality which the participant is willing to watch. The maximum rate may be derived from the display equipment limitations. Destination may also restrict the total received throughput from all of the sources. The following sections focus on the destination fairness problem where each destination may only specify a firm ratio between the sources. The network management target is to maximize the total capacity allocation of the destinations under the strict ratio constraint, while utilizing the Max-Min fairness principle among the destinations.

The Network Model

Our network is modeled as a general undirected graph $G(V,E)$. Each network component, such as a switch and a router, is represented by a vertex. We assume that vertices cannot fail or come up during the algorithm. $D \subseteq V$ is the destinations group. The edge $(i,j) \in E$ represents a link between two vertices of the network and C_{ij} represents the edge free capacity (available for this particular conference). We assume a reliable communication between the vertices, i.e., each message reaches its destination within a finite time. R_{sd} is an ordered set of edges that represents a single routing path from source s to destination d . Each destination may receive flow from many other sources over such routing paths. f_{sd} is the flow from source s to destination d . Every destination d determines for each source s the proportion a_{sd} . a_{sd} is the proportion of source s flow in destination d total flow f_d , i.e., $f_{sd} = a_{sd} * f_d$ and $\forall d \in D \sum_{s \in V} f_{sd} = f_d$. Clearly $\forall d \in D \sum_{m \in V} a_{md} = 1$. The flow allocations are represented by a sorted flow vector $\vec{F} = \{f_{d_1}, f_{d_2}, \dots\}$, where f_{d_1} is the smallest allocated flow.

The Extended Max Min Fairness Criterion- (EMMF)

Allocating fairly a limited capacity resource among participants requires a definition of fairness. The question is how to allocate fairly the conference capacity according to the destinations proportion requirements. The most common definition of fairness is the Max Min Fairness (MMF) [4]. The MMF solution is used in [5] for flow control fairness to provide for each flow f_{sd} a fair share. We use the formal

known definition at [6] and present an extension to our conference model.

Let $\vec{F} = \{f_{d_1}, f_{d_2}, \dots, f_{d_m}\}$ and $\vec{F}^* = \{f_{d_1}^*, f_{d_2}^*, \dots, f_{d_m}^*\}$. We say that $\vec{F}^* \leq \vec{F}$ if vector \vec{F}^* is lexicography equal or smaller than vector \vec{F} . Alternatively $\vec{F}^* > \vec{F}$ if $f_{d_1}^* > f_{d_1}$ or if $\forall j < l$ $f_{d_j}^* = f_{d_j}$ then $f_{d_l}^* > f_{d_l}$ for some $l \leq m$.

Vector \vec{F} is feasible, if and only if $\forall (i, j) \in E \sum_{\{s, d | (i, j) \in R_{s, d}\}} f_d * a_{sd} \leq C_{ij}$

The vector $\vec{F}^ = \{f_{d_1}^*, f_{d_2}^*, \dots\}$ is a Max – Min fairness vector, if and only if it is feasible, and for each feasible \vec{F} $\vec{F} \leq \vec{F}^*$.*

As already noted in the literature [6], the MMF approach leans heavily towards equal allocations of the resources, sometimes at the expense of system efficiency. In Appendix A we suggest a new criterion that relaxes the fairness constraint towards a maximum destination total throughput.

3.1 The Extended Max-Min Fairness Algorithm (EMMF)

Variables used by the algorithm are as follows. d - The group of destinations without capacity allocation. c_{ij} - Free residual capacity of edge (i,j). r_{ij} - The current allocated Max flow per destination in d , computed at edge (i,j).

Outline : The central algorithm computes the destination allocation vector that is EMMF optimal. It is assumed that the algorithm is provided with all the necessary network information such as routing paths and edges capacity. The first stage of the algorithm initializes the variables. Each edge (i, j) residual capacity c_{ij} is set to be C_{ij} the capacity of the edge. Group d , the destinations without capacity allocation, is initialized to D the group of all the destinations. The algorithm starts with calculating the maximum destinations flow allocations allowed by each edge r_{ij} , under the constraint of equal destination flow to all the non-allocated destinations (all the vertices in group d). Each edge (i,j) has its own limitation $r_{ij} = \frac{c_{ij}}{\sum_{\{m, n | (i, j) \in R_{m, n} \text{ \& } n \in d\}} a_{mn}}$.

This means that for each flow which passes through edge (i,j) and its destination is a member of d , it allocates the maximum available flow, taking into account the local proportion of these flow allocations and the edge's residual capacity. The chosen

flow is the smallest r_{ij} flow for all edges (i,j). This flow can be allocated to all the non-allocated destinations without overflowing any edge in the network. The chosen edge (m,n) is the current bottleneck edge of the network. The destinations that route their flow through the bottleneck edge (all k such that (m,n) $\in R_{vk}$) are clearly limited by the flow allocated at this edge (r_{mn}). Therefore, r_{mn} is allocated to every such limited destination k ($f_k = r_{mn}$), and its allocated flow is subtracted from the residual free capacity of all the edges that it traverses. These participants end their role as destinations in the algorithm, and they are removed from the group of the non-allocated destinations d . The algorithm described above is repeated as long as d is not empty and uses the new values of d and c_{ij} .

Theorem 1 *The flow allocation vector \vec{F} , the result of the EMMF algorithm, is an EMMF vector.*

A formal statement of this algorithm along with a formal proof of Theorem 1 are presented in [7].

Example:

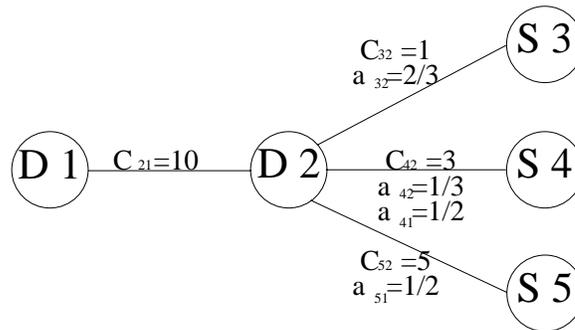


Figure 1: Optimal algorithm example

Figure 1 depicts the example graph where $V = \{1,2,3,4,5\}$, $E = \{(2,1),(3,2),(4,2),(5,2)\}$ with 5 vertices all of which are participants. The capacities of the edges are $C_{21} = 10$, $C_{32} = 1$, $C_{42} = 3$, $C_{52} = 5$. The routing path from sources to destinations are

$R_{41} = \{(4, 2), (2, 1)\}$ $R_{51} = \{(5, 2), (2, 1)\}$ $R_{32} = \{(3, 2)\}$ $R_{42} = \{(4, 2)\}$. Destination 1 determines the proportions to be $a_{41} = 1/2$, $a_{51} = 1/2$ and Destination 2 determines the proportions to be $a_{32} = 2/3$, $a_{42} = 1/3$.

The algorithm is conducted as follows. The free capacity of each edge is set to its capacity $c_{21} = 10$, $c_{32} = 1$, $c_{42} = 3$, $c_{52} = 5$. $D = \{1,2\}$ has only two destinations, Participant 1 and Participant 2. The first calculation of r_{ij} results in $r_{21} = \frac{10}{1/2+1/2}$, $r_{32} = \frac{1}{2/3}$, $r_{42} = \frac{3}{1/3+1/2}$, $r_{52} = \frac{5}{1/2}$. In order to identify the bottleneck, the edge with the smallest r_{ij} is chosen. In this case, edge (3,2) is chosen as the first bottleneck and the minimum flow is $r_{32} = 3/2$. The only path that goes through edge (3,2) is the routing path between Source 3 and Destination 2. Destination 2 flow is determined to be $f_2 = 3/2$. The flow from Source 3 to Destination 2 is $(3/2)*(2/3)=1$, and the flow from Source 4 to Destination 2 is $(3/2)*1/3=1/2$. This allocated capacity is subtracted from the edges leading flow to Destination 2. $c_{32} = 1 - 1 = 0$, $c_{42} = 3 - 1/2 = 5/2$, $c_{52} = 5$. Since Destination 2 received its capacity allocation it is removed from the non-allocated group $d = \{1\}$.

Group d is still not empty so the algorithm is repeated from the first stage and calculates r_{ij} as $r_{21} = \frac{10}{1/2+1/2}$, $r_{32} = \infty$, $r_{42} = (5/2)/(1/2)$, $r_{52} = 5/(1/2)$. In this case we choose edge (4,2) as the bottleneck and the minimum flow is $r_{42} = 5$. The only flow that goes through the edge (4,2) is via the path between Source 4 and Destination 2. The second path that goes through the edge (4,2) has already been allocated. The current allocated destination is 1. Destination 1 flow is determined to be $f_1 = 5$. Capacity is allocated to all the paths leading to it. The flow from Source 4 to Destination 2 is $5*(1/2)=5/2$, and the flow from Source 5 to Destination 1 is $5*(1/2)=5/2$. All the allocated capacity is removed from the edges, $c_{21} = 10 - 5 = 5$, $c_{42} = 5/2 - 5/2 = 0$, $c_{52} = 5 - 5/2 = 5/2$. Since capacity was allocated to destination 1 it is removed from the non-allocated group d . Group d is finally empty and all the destinations have received their allocations leading to the end of the algorithm. The result of the algorithm is the EMMF allocation vector $\vec{F} = \{f_2 = 3/2, f_1 = 5\}$.

3.2 The Distributed Algorithm

Outline : The distributed algorithm computes the EMMF flow vector. The algorithm is composed of three algorithms: the source algorithm, the destination algorithm and the edge algorithm. In practice the (i,j) edges algorithm is executed at vertex i . The algorithm uses a single message format. The message includes a source identifier, a destination identifier, a minimal flow and a final mark. It is assumed that the algorithm starts simultaneously at all the destinations. Later we discuss how this requirement is relaxed.

The algorithm progresses by phases. At each phase, the destinations send the current chosen minimal flow to all their sources. All the edges over the flow paths pass this information unchanged and with no delay. At the first phase, the chosen minimal flow is set to zero and marked as “not final”. The sources respond to each destination message with a similarly constructed message that is sent back over the path toward the destination. At the first hop, this message carries an infinite value for the minimum flow. Before handling a source oriented message, each edge in the routing path waits until it receives all the current phase messages of the chosen minimal flow allocation (over the opposite direction) from all the destinations whose routing path goes through it. When each edge receives all such messages, it calculates its local minimal flow for that phase. The edge flow calculation is EMMF as was described for the centralized algorithm. Each edge transmits the minimum between the minimal flow that it receives from the nearest edge (down stream from the source), and its local calculated minimal flow. Each edge has its own unallocated group, a group of all source and destination pairs that do not receive their allocation. At the beginning of the algorithm this group at each edge is initiated to all the pairs that have a routing path through this specific edge. An edge marks its flow as “final” if all of its unallocated destinations choose the edge local minimum flow as their chosen minimal flow for that phase. Otherwise, the edge marks its MMF allocation as non-final. At the end of each phase, the destination receives the minimal flow over all the routing paths and chooses the minimum flow between all the paths. At this point it starts a new phase by sending this value to all its sources including the chosen flow

marked as final or non-final.

Each edge in the routing path subtracts the destination chosen minimal flow from the edge capacity only when the chosen flow is marked final, i.e., it is not going to be changed. In these phases the algorithm determines the flow allocations of the next bottlenecks, similar to the centralized algorithm described in Section 3.1. A bottleneck is found when an edge marks its flow as final. The algorithm terminates at a destination when its flow is going through a bottleneck edge. It receives a final status message and forwards it to all of its sources. It terminates at an edge when its unallocated group is empty, i.e., it receives a final marked flow message from all the destinations passing through it. It terminates at a source when it receives a final message from all its destinations. A formal statement of this algorithm along with a formal proof of its correctness and main properties are presented in [7].

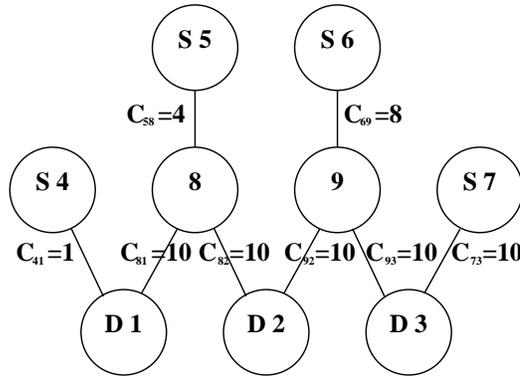


Figure 2: distributed algorithm example

Distributed algorithm example: Figure 2 depicts the example graph where vertices $\{1,2,3\}$ are the destination group, and vertices $\{4,5,6,7\}$ are the source group. Each destination has two sources with equal proportions. Destination 1 has Source 4 and Source 5. Their proportions are $a_{41} = a_{51} = 1/2$. Destination 2 has Source 5 and Source 6. Their proportions are $a_{52} = a_{62} = 1/2$. Destination 3 has source 6 and source 7. Their proportions are $a_{63} = a_{73} = 1/2$. The capacities of the edges are $C_{81} = C_{82} = C_{92} = C_{93} = C_{73} = 10$ and $C_{41} = 1$, $C_{58} = 4$, $C_{69} = 8$.

The routing paths from sources to destinations are the shortest paths. For example $R_{41} = \{(4, 1)\}$, $R_{51} = \{(5, 8), (8, 1)\}$. The algorithm is conducted as follows: the first phase starts when all the destinations transmit their minimal chosen flow allocations set to zero. All the edges in the reverse routing paths pass this message toward the destinations without any change or delay. For each message received by the sources, each source reacts by sending a message which allows an infinite flow toward the received message specific destination. The edges in the routing path from source to destination wait until they receive all the messages from their destinations. In this phase they are all zeroes, and then calculate their MMF allocations. For example, observe the routing path $\{(5,8),(8,1)\}$. Edge (5,8) receives from its two destinations $\{1,2\}$ their minimal chosen flow 0. When it receives a message that allows an infinite flow, from Source 5 to Destination 1, it passes the minimum between its calculated minimal flow 4 and the infinite flow. Edge (8,1) does the same when it receives the minimal flow from edge (5,8). It passes the minimum between its minimal calculated flow 5 and the received minimal flow 4. Destination 1 receives a minimal flow message equal to 4 on its right edge. In the second phase each destination chooses the minimum between the received flow allocations and passes it toward its sources. Destination 1 chooses 2 as the minimal flow and sends it towards sources $\{4,5\}$. In this phase, the first bottleneck is found. Edge (4,1) receives a minimal chosen flow from destination 1 equal to its allocated flow. Therefore the edge marks this flow as final. In the third phase destination 1 forwards its final flow over all its routing paths. Edge (8,5) marks Destination 1 flow as final and increases Destination 2 allocation to 6. This is repeated until all the destination flow allocations are marked final. The result of the algorithm is the EMMF allocation vector $\vec{F} = \{f_1 = 2, f_2 = 6, f_3 = 10\}$.

The Algorithm Main Characteristics

The algorithm avoids redundant calculations at the edges. Each edge waits for all the chosen minimal flow messages from all the routing paths that pass through it. When it has the full knowledge of all the chosen minimal flow allocations, it calculates its minimal flow for the current phase. The algorithm computation complexity is

equal to the centralized algorithm. The method of spreading the information in phases reduces the number of messages that are sent in the algorithm, reducing the message complexity. The algorithm has a “fast start” feature, i.e., it allows the source to start transmitting data at the first stage. In other words, when a source receives the first chosen minimal flow message it can start transmitting. This is safe in terms of available bandwidth, as it is known that this flow is smaller or equal to its final (yet unknown) flow. Therefore, the sources do not overflow the edges even if they start their transmissions as soon as possible.

In the distributed algorithm we assumed that all the destinations start the algorithm at the same time. In order to relax this restriction the trigger for starting the algorithm can be performed using a Propagation of Information (PI) [8] algorithm, i.e., starting the algorithm at one vertex that floods a START message to all the destinations. Each destination starts the EMMF algorithm when it gets the START message. This change in the initializing conditions does not change the algorithm. A formal proof of this characteristic is presented in [7].

4 Time Resource Management - (TRM)

Implementing video conferencing over a communication network enables the control and sharing of the time resource. The Time Resource Management (TRM) point of view is a central one. It collects the timing information and requirements from all the conference participants and forces timing policies upon them. The TRM central position along with its extended communication, computation and control abilities enables it to manage the time resource better than a human moderator. It should result in time management which is more efficient (waste less time on management and better prevent collisions), precise and easy to use. In order to satisfy the above objectives, the TRM should be aware of the participants’ demands. These demands can be divided into a priori and interactive demands. A priori demands are presented by participants before they are admitted to the conference. The interactive demands are introduced during the conference and present the current interactive participant demands. In order to satisfy these demands using a scheduling algorithm, they are

presented to the TRM in terms of QoS constraints. The TRM algorithm also depends on the conference model. Several conference models were described in section 2, each having its own time management constraints. The objectives of the next sections are to define the main participant QoS time constraints and to map them to known QoS-based scheduling problems and solutions that were developed for other systems.

4.1 Participants Time Requirements

As described above, participants needs must be defined and specified in terms of QoS constraints. There are many types of participant applications such as real-time video, video clips, broadcast audio and human speakers. Each application may have specific QoS constraints. This section focuses on the timing requirements of human speaker participants. We present these needs in a formal and quantitative way. As mentioned before, these requirements can be divided into a priori and interactive requirements. Let us start with the a priori requirements. A participant may require to speak for a time that is at least a portion of the total conference time. A participant may require to limit his waiting time from the time that he asks to speak to the time that he gets his next speaking right. Each time a participant speaks he may require not to be preempted for at least a minimum period of time before he loses his turn. Before the speaker is preempted, he may wish to be alerted a specified period of time before the preemption. In conferences where participants can speak simultaneously, it may be necessary to limit the maximum number of participants speaking simultaneously at each speech level.

Although these QoS time constraints are a priori requirements, they may still be time dependent. There are cases for instance, where the opportunity to speak becomes more important as one reaches the end of the conference. At such times, more participants may want to be granted speaking rights.

We focus on what we consider to be the participant main a priori QoS time requirements. We specify them for a given speaker i .

- 1) ϕ_i - The minimum fraction of speaker i 's total speaking periods from the sum of its waiting periods (times where i wishes to speak but has to wait) and its speaking

periods. Note that if i is greedy, i.e., wishes to speak as much as possible, his waiting periods and his speaking periods are equal to the conference duration.

- 2) L_i - The maximum delay from the time that a participant requests to speak till the time that he gets his speaking right.
- 3) P_i - Minimum speaking period without preemption.
- 4) M - Maximum number of participants speaking simultaneously.

We now turn to interactive requirements. A participant may also present timing requirements during the conference. He may submit a request for speaking rights. When he does, he may want to specify its characteristics. It could be a remark (a short request of high priority), or a speech (a request for a longer time). He may always stop speaking before the end of his requested speaking period. A participant may give up his time share for the sake of other participants. He may want to affect TRM decisions by presenting his preferences regarding his favorite speaker. The TRM may allocate time according to the participant's interactive rating, i.e., a participant with high rating can get a longer speaking time than a participant with a low participant rating.

We focus on two main interactive signals. 1) Requesting speaking permission. 2) Signaling the end of speech. We will later extend these requirements.

4.2 The TRM Problem

The TRM input is the participants' QoS time requirements, the conference model, and the conference timing. The TRM is composed of the TRM admission control management and the interactive management. The interactive management is responsible for managing the conference time resource in real-time, respective to the a priori parameters and the interactive parameters. The admission control management is responsible for checking whether the a priori input parameters are feasible or not, i.e., it checks if the scheduling algorithm can fulfill all the time constraints in the worst scenarios. Let us summarize the input parameters of the TRM considered in this section.

* The conference total time is T_{total} .

* The total number of participants is N .

* Participants may not speak simultaneously.

* The participants have a priori QoS constraints summarized in the vectors:

$$\vec{\phi} = \{\phi_1, \phi_2, \dots, \phi_N\}, \vec{P} = \{P_1, P_2, \dots, P_N\}, \vec{L} = \{L_1, L_2, \dots, L_N\}.$$

* The participants have the following interactive requests: They can ask for speaking permission. They can stop speaking before the termination of their allocated period.

In order to solve this TRM problem we map it to another known scheduling problem with a few changes. Our reference scheduling problem is the one that addresses link scheduling for contending packets, (see [9, 10, 11]). Here we focus on a particular solution termed the Weighted Fair Queuing (WFQ) [11]. The WFQ policy is a modification of the Generalized Processor Sharing (GPS), and is also termed PGPS (Packet Generalized Processor Sharing) [10].

In the following, we briefly review the results of [10]. The GPS server serves packets from N different sessions. Each packet has an arbitrary length with a maximum of P_{max} . The server operates at a fixed rate $r = 1$. GPS is a work-conserving server, i.e., it must serve if there are packets waiting for service. Each session i is characterized by a positive number ϕ_i such that $\sum_j \phi_j \leq 1$. A session is backlogged if there are packets waiting to be processed. Let $S_i(\tau, t)$ be the amount of session i traffic served in an interval (τ, t) , and let $B(t)$ be the group of sessions being served under GPS at time t . The GPS provides for any backlogged session i in time interval of (τ, t) a fraction ϕ_i such that $\frac{\phi_i}{\phi_j} \leq \frac{S_i(\tau, t)}{S_j(\tau, t)}$ $j = 1, 2, \dots, N$. At any given time t , session i is guaranteed a rate of $g_i = \frac{\phi_i}{\sum_{j \in B(t)} \phi_j}$. The GPS assumes that the server can serve multiple sessions simultaneously and that packets are infinitely divisible. The PGPS is a packet-by-packet processing scheme. Let F_p be the time at which packet p departs under GPS. PGPS is also a work-conserving server which serves packets in increasing order of F_p . Further explanations and examples are given at [10] and [11].

This scheduling policy is mapped to the current TRM problem. The participants speaking period portion is denoted by the minimum guaranteed rate ϕ_i . Participant

i 's request for a speaking period is mapped to an arrival of a packet from session i whose length is the requested period of time P_i . The PGPS problem as defined in [10] and the TRM model are slightly different. In the PGPS definition the scheduling algorithm actions do not affect the packets arrival process. In the TRM problem definition the scheduling algorithm has a direct effect on the participants speaking requests. The participant is not allowed to place a request for another speaking period before he is satisfied with his current request. Denote by $F i_p$ and $F' i_p$ the time at which packet p of participant i finishes service under GPS and PGPS, respectively. In the case where PGPS ends serving a packet after GPS ($F i_p < F' i_p$) and the specific participant i requests another speaking period of time immediately after finishing his service under PGPS ($F' i_p$), it is assumed that the arrival time of this request is the time at which his previous request finishes its service under the GPS server ($F i_p$). Another difference is that the PGPS algorithm cannot guarantee a maximum delay between two consecutive speaking periods of a greedy participant (L_i), i.e., it cannot guarantee a maximum delay between the time when a participant asks to speak and the time when he starts speaking. This characteristic is described in [9].

Therefore, we solve the TRM scheduling problem using another scheduling algorithm termed the WF²Q policy. The WF²Q policy operates like PGPS with the following difference. It schedules the next request with the smallest F_p (of the GPS) only among those packets that commenced service under GPS. $F^* i_p$ denotes the time at which packet p of participant i terminates service under WF²Q. Note that the only difference from PGPS is when WF²Q terminates serving a packet before GPS, $F^* i_p \leq F i_p$, and when the participant i next request ($p + 1$) arrives before $F i_p$. In this case PGPS can serve this request but, under WF²Q, this packet can be served only after $F i_p$.

As mentioned before, the TRM admission control should check whether the conference time constraints are feasible or not. Under the WF²Q policy the TRM admission control should check for each participant i if $L_i \leq \frac{2 * P_i}{\phi_i} + P_{max\tau} - P_i$ where $P_{max\tau}$ is the maximum speaking duration without preemption of all the participants except i . Using the technics of [9] the above property is proved in appendix B. Under this

condition the WF²Q can fulfill all the TRM QoS demands.

4.3 Practical Extensions

The functions of the TRM can be expanded. The TRM can inform the participant that he is going to be preempted, at an agreed period of time (AL_i) before the preemption. This can be implemented in the WF²Q based TRM, by informing the current speaker AL_i time before the expiration of P_i . If the current speaker is the only participant that asks to speak, he can continue speaking until the arrival of the first request. When such a request appears, the current speaker is alerted and may continue speaking only for a period of AL_i . Note that this change does not affect the value of L_i .

The set of interactive requests can be expanded too. Instead of applying for a fixed a priori non-preempted period P_i , a participant can request $P'_i \leq P_i$ for the next speaking period. The WF²Q schedules participant's requests according to their termination time under GPS. By applying for a shorter period of time, the participants get shorter waiting periods. Therefore, short messages such as remarks are served before long messages such as speeches. The TRM can also limit the total number of speaking periods of each participant. A priori, each participant can be limited to n_i speaking periods at the conference. This limitation prevents participants from applying too frequently for short speaking periods. By determining $n_i = \frac{\phi_i * T_{total}}{P_{ev}}$, the greedy participant's average speaking period is P_{ev} .

The speaking portion of each participant ϕ_i and his non-preempted time period P_i can be interactively changed by the TRM. As mentioned before, the participant's speaking portion can be changed by his rating. If a participant has a high rating, he may receive a longer speaking time by increasing his ϕ_i interactively. The flexibility of the algorithm enables the schedule policy to be affected by the remaining conference speaking-time. The non-preempted speaking period can be decreased by the end of the conference. It can also overcome the following problem that is related to the end of the conference. Towards the end of the conference, the remaining conference time may be smaller than the non-preempted periods of the current waiting participants. If such

a problem arises the TRM may reduce the participants non-preempted time period according to the residual time. For example, if T_r is the conference residual time, the TRM computes new speaking periods $p'_i(t)$ such that $P'_i(t) = \text{Min}\{P_i, \frac{T_r * \phi_i}{\sum_{j \in B(t)} \phi_j}\}$.

In summary, defining participant needs as QoS constraints maps the TRM objective to the known general area of time scheduling. A rich variety of time scheduling policies can be found in the literature, in fields such as computer networks and operating systems. Some of these policies can be adapted to solve the TRM problem and yield satisfactory solutions. The TRM problem has unique characteristics which call for some adaptations in the selected solutions.

5 Video Conference Management - Simple Examples

In this section we sketch a high level implementations for several video conferencing models. We first describe simple participant computer's environment (Desktop Video Conferencing). The DVC screen presents the designated participants' pictures. Beside the current speaker, there is a graphical time indicator which indicates how much time he has to speak before being preempted, and the promised alert time before preemption. Another indicator denotes the residual waiting time before the speaking permission of the DVC's user. The DVC's user applies for a speaking period by using a designated DVC's button. This period $P'_i \leq P_i$ can be changed interactively by the participant. The participant can terminate speaking by pushing another button.

Implementing video conference management demands an integrated management of time and capacity. This section focuses on the combination of the TRM and the CRM. It suggests generic implementations for the models which are described in Section 2. Each of these models demands different interaction between the TRM and the CRM. In each of these models, a participant who is admitted to the conference determines his capacity demands and his time demands. The conference management starts by operating the admission control of both the TRM and the CRM. The admission controls can interactively offer available and feasible parameters, or can analyze the participants' requests. The admission is allowed only if the participants'

demands are feasible.

The Peer Meeting Model: As mentioned before, in this model participants play a similar role but may have a different degree of information to present. For simplicity, we assumed that the number of participants is limited in order to allow simultaneous presentation of their figures on each participant screen. Under this assumption it is logical to operate TRM and CRM consecutively. Before the beginning of the conference the CRM allocates fixed capacity allocations to the participants. Throughout the conference the TRM allocates time interactively according to the participant demands. With this implementation each participant can watch all of the participants simultaneously on his screen and listen only to the current speaker (at the high speaking volume).

The Lecture Model: This model has one lecturer and a large audience. There is a definite difference between the lecturer's QoS requests and the audience's requests. In this model it is not practical to watch all of the audience members simultaneously on a DVC screen. Throughout the conference the lecturer's image appears on each participant's DVC screen. The image of the participant asking a question appears only when he speaks and there is only one participant speaking at a time. This mechanism calls for an interaction between the TRM and the CRM. The CRM has two main states, (i) the lecturer speech, and (ii) audience questions. When the lecturer is speaking the CRM allocates capacity only to the lecturer traffic (which includes his image and his multi-media facilities). Each time the TRM decides to schedule one of the audience it informs the CRM to allocate capacity to both the lecturer and the speaker flow. In the TRM implementation the lecturer's parameters (P_1 and ϕ_1) are high, and L_i is low. The audience parameters (P_i , ϕ_i) are low and L_i is high. There are only two request queues: the lecturer queue, and the audience queue. Both queues are managed FIFO. We assume that most members of the audience remove their requests while listening to other participants from the audience.

The Multi - group Model: The multi-group conference consists of a number of participants groups. Each participant can take part in more than one group. When a participant wishes to speak he determines his destination group, interactively. For

example, at the lecture model, some of the audience belonging to the same group can speak among themselves without interrupting the lecturer or other participants. To manage such a conference we operate the CRM before the beginning of the conference. The CRM allocates capacity fairly without relating to the group division. The multi-group model is characterized by operating a number of TRM algorithms simultaneously. Each participant takes part in the TRM algorithm of each one of his groups. In the above example, each participant is involved with the lecture TRM and with his chat group TRM.

6 Summary

A comprehensive approach to the management of large scale multimedia conferencing is presented. We first identify the need to manage the different conference resources and share them effectively among competing participants. We also need to monitor and control the amount of participants traffic in order not to overflow the limited resources.

Our solution divides the resources into capacity and time resources which can also be related to destinations and sources demands respectively. We focus on devising fair solutions for both problems. For the capacity allocation we extend the traditional Max-Min fairness approach to include relations between the requested sources. For the time management we show an analogy between this problem and other scheduling problems and adopt known results to our needs.

Our solutions and observations should only serve as a preliminary study of the larger problem space. As explained in the various sections, other models and solutions should be considered. We believe that much more work is needed for a deeper understanding of this problem and for its efficient solution.

Appendix

A Delta Expanded Max-Min Fairness

General

In the previous sections the main objective of the CRM algorithms was to achieve fair capacity allocation between the conference participants. This objective is achieved using the EMMF criterion that suggests a specific solution to the fair capacity allocation problem. There are cases where a small deviation from the fair allocation significantly increases the overall destinations flow. For instance, a small decrease in a low density flow can cause a big increase in a higher density flow. Clearly, a decrease in a high flow density can never cause an increase in a lower density flow in the EMMF flow vector. This can be seen in the following example in Figure 3.

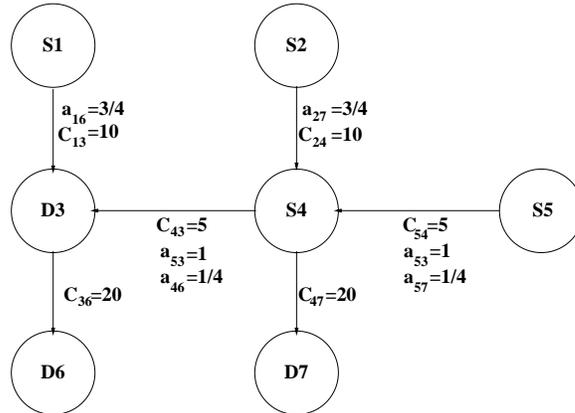


Figure 3: Delta Max-Min fairness example

Delta Max-Min fairness example : Destination 3 receives flow from Source 5. Destination 6 receives flow from Source 1 and Source 4 with the following proportions $a_{16} = 3/4$, $a_{46} = 1/4$. Destination 7 receives flow from Source 2 and Source 5 with the following proportions $a_{27} = 3/4$, $a_{57} = 1/4$. The edge capacities are $C_{13} = C_{24} =$

10, $C_{43} = C_{54} = 5$, $C_{36} = C_{47} = 20$. The resultant vector of the EMMF algorithm is $\vec{F}_1 = \{f_3 = 4, f_6 = 4, f_7 = 4\}$ with a destinations flow sum of 12. If the allocated flow changes, even by one flow unit, the flow vector will be $\vec{F}_2 = \{f_3 = 3, f_6 = 8, f_7 = 8\}$ with a destinations flow sum of 19.

Such an improvement can also be achieved when the conference includes a destination flow which has a very small proportion coefficient at its bottleneck edge and enough free capacity at the other edges that route the flow to the same destination. These characteristics can be seen in the example. The connection from Source 5 to Destination 3 shares two connections in its routing path in separate edges. Their proportion coefficients are small and there is enough free capacity in the edges to route the flow to Destinations 6 and 7. In order to improve the overall destinations flow in such cases, without sacrificing the fairness property too much, we introduce a new class of algorithms termed $\vec{\delta}$ EMMF.

The $\vec{\delta}$ EMMF Criterion definition

Throughout these examples it can be seen that a small deviation from the fairness constraint can increase the destinations total throughput significantly. The deviation in the fairness constraint is defined as a proportional distance from the optimal EMMF vector.

Let $\vec{F}^* = \{f^*_{d_1}, f^*_{d_2}, \dots, f^*_{d_N}\}$ be the EMMF vector. Let $\vec{\delta} = \{\delta_1, \delta_2, \dots, \delta_N\}$ be the proportions vector st. for each i $0 \leq i \leq N$ $0 \leq \delta_i \leq 1$. Vector $\vec{F} = \{f_{d_1}, f_{d_2}, \dots, f_{d_N}\}$ is $\vec{\delta}$ fair vector, if and only if it is feasible (as defined in 3), and for each $d \in D$ (the destinations group) $\delta_d * f_d^* \leq f_d$.

The vector $\vec{F}' = \{f'_{d_1}, f'_{d_2}, \dots, f'_{d_N}\}$ is $\vec{\delta}$ EMMF vector if and only if it is $\vec{\delta}$ fair vector, and for each $\vec{\delta}$ fair vector \vec{F} $\sum_{k \in D} f_K \leq \sum_{k \in D} f'_K$

The capacity resource management problem is to find the $\vec{\delta}$ EMMF vector and to allocate capacity respectively.

The $\vec{\delta}$ EMMF as a Linear Programming Problem

This problem is a standard linear programming problem of maximum type as defined at [12]. The objective is to maximize the overall destinations flow, $\sum_{k \in D} f_k$ subject to the capacity restrictions of each edge. The sum of destinations flow at each edge must be less equal than its capacity. For all $(i,j) \in E$ $\sum_{\{m,n|(i,j) \in R_{mn}\}} a_{mn} * f_n \leq C_{ij}$, and they are subjected to the fairness restrictions which limit the distance from the optimal EMMF vector. For all $k \in D$ $(1 - \delta_k) * f_k^* \leq f_k$, and $0 \leq f_k$. We present this problem in formal way as the standard form of linear problem:

$$\text{For all } (i,j) \in E \quad X_{ij} + \sum_{\{m,n|(i,j) \in R_{mn}\}} (a_{mn} * f_n) = C_{ij}, \quad 0 \leq X_{ij}.$$

$$\text{For all } k \in D \quad f_k - Y_k = (1 - \delta_k) * f_k^*, \quad 0 \leq f_k, \quad 0 \leq Y_k.$$

X_{ij} and Y_k are slack variables. In this problem, the simplex is a bounded region and the solution is at one of the simplex vertices. Let n be the number of edges and N the number of destinations, the number of the unknown variables is $n+2N$. There is a total of $n+N$ equations. The linear problem pivoting solution of the equations is achieved by setting N variables to zero.

If we look at the algorithm results on the given example 3 while $\vec{\delta}=\{3/4,1,1\}$ we see that at first the optimal EMMF algorithm from section 3.1 should be executed, and the EMMF vector is $\{f_3 = 4, f_6 = 4, f_7 = 4\}$. as mentioned before. The result of the linear programming algorithm is $\{f_3 = 3, f_6 = 8, f_7 = 8\}$. This is the $\vec{\delta}$ EMMF vector with the maximum destinations total throughput.

B Time Resource Management - (TRM)

B.1 PGPS maximum delay

The PGPS schedule algorithm cannot guarantee a maximum delay between two consecutive speaking periods of a greedy participant (L_i), i.e., it cannot guarantee a maximum delay between the time when a participant asks to speak and the time at

which he actually starts speaking. This characteristic is described in Figure 4. Here, there are $N = 6$ greedy participants at the conference. Their minimum speaking periods are equal, $P_i = 1 \forall i$. Their ϕ_i 's are described in the figure. As we can see, the delay period between two speaking periods of participant 1, period e and period k, is five units. It can be increased to ten units by adding, for instance, five more greedy participants, and changing ϕ_i 's to $1/20$ for all $2 \leq i \leq 11$. The delay between two consecutive speaking periods of participant no.1 is effected by the number of other participants and their value of ϕ .

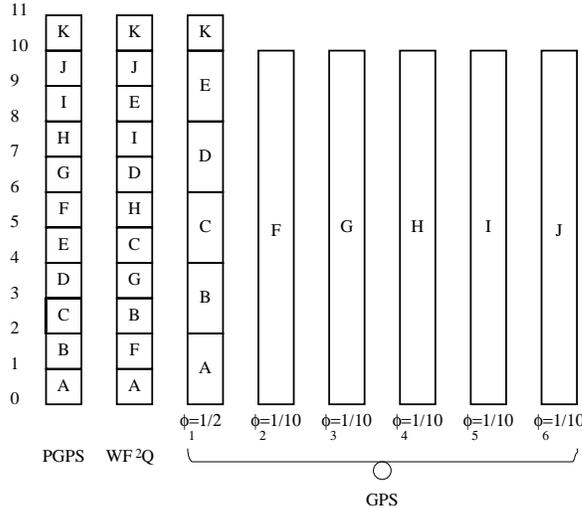


Figure 4: *PGPS, WF²Q, GPS* example

B.2 WF^2Q maximum delay

Let F_p and F_p^* be the time at which packet p departs under GPS and WF^2Q , respectively.

Lemma 1

$$F_p - F_p^* \leq \frac{P_i}{\phi_i} - P_i$$

The proof of this lemma is given in [9]

Let $P_{max_{\bar{i}}} = \text{Max}_{j \neq i} \{P_j\}$. Let S^*i_p and Si_p be the starting time of participant i packet p under WF^2Q and GPS, respectively. Let F^*i_p and Fi_p be the time at which packet p of participant i departs under WF^2Q and GPS, respectively. Let Ai_p be the time at which packet p of participant i arrives.

Theorem 2

$$S^*i_p - F^*i_{(p-1)} \leq \frac{2*P_i}{\phi_i} + P_{max_{\bar{i}}} - 2 * P_i \text{ s.t } Ai_p \leq F^*i_{(p-1)}$$

Proof

Participant i finishes his $p - 1$ speaking period at $F^*i_{(p-1)}$. This means that the earliest time he may start speaking is P_i time before that, i.e., $F^*i_{(p-1)} - P_i \leq S^*i_{(p-1)}$. Since WF^2Q schedules only these packets that already started their service under GPS, packet $p - 1$ latest service starting time under GPS is its starting time under WF^2Q , i.e., $Si_{(p-1)} \leq S^*i_{p-1}$. Packet $p - 1$ latest departure time under GPS is $\frac{P_i}{\phi_i}$ after $Si_{(p-1)}$. Hence, $Fi_{(p-1)} \leq \frac{P_i}{\phi_i} + Si_{(p-1)}$. Since $Ai_p \leq F^*i_{(p-1)}$, the latest time that packet p may start service under GPS is $Fi_{(p-1)}$, and the latest time it departs under GPS is $Fi_p \leq \frac{2*P_i}{\phi_i} + S^*i_{(p-1)}$. In [7] it is concluded that the latest departing time of packet i_p under WF^2Q is $P_{max_{\bar{i}}}$ after Fi_p . Therefore packet i_p latest service starting time at WF^2Q is $S^*i_p \leq \frac{2*P_i}{\phi_i} + S^*i_{(p-1)} + P_{max_{\bar{i}}} - P_i$. Therefore $S^*i_p - F^*i_{(p-1)} \leq \frac{2*P_i}{\phi_i} + P_{max_{\bar{i}}} - 2 * P_i$

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